SCOTCH COLLEGE Mathematics Department	**Scotch College** Mathematics Specialist Test One -SOLUTIONS Date: 3rd December 2015

NAME:____

Time allowed

Section	Reading	Working
Calculator-free	2 minutes	25 minutes
Calculator-assumed	2 minutes	25 minutes

Section One (Calculator-free): 27 marks

Permissible items:

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet

Write your answers in the spaces provided.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use** pencil except in diagrams.

	Question	Marks available	Marks awarded
Calculator Free	**1**	**6**	
	2	**6**	
	3	**10**	
	4	**5**	
	Section One Total	**27**	
Calculator Assumed	**6**	**7**	
	7	**4**	
	8	**7**	
	9	**7**	
	Section Two Total	**25**	
	Total:	**52**	

Section One: Calculator-free **[27 marks]**

This section has **Four (4)** questions. Answer **all** questions. Write your answers in the spaces provided

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Question 1 [6 marks]

Simplify each of the following expressions, writing your answer in exact polar form.

(a) $\left(\sqrt{3}-i\right)^2\left(\sqrt{3}-i\right)$

$$=\left(\sqrt{3}-i\right)^3$$

$$=\left[2cis\left(\frac{-\pi}{6}\right)\right]^3$$ ✓ change to polar form

$$=8cis\left(\frac{-\pi}{2}\right)$$ ✓Applies de Moivre's theorem

[2]

(b) $3cis\left(\frac{\pi}{4}\right)\times\left[2cis\left(\frac{-\pi}{3}\right)\right]^{-1}$

$$=3cis\left(\frac{\pi}{4}\right)\times\frac{1}{2}cis\left(\frac{\pi}{3}\right)$$ ✓Applies de Moivre's theorem

$$=\frac{3}{2}cis\left(\frac{7\pi}{12}\right)$$ ✓Simplifies answer

[2]

(c) $\dfrac{1}{\sqrt{2cis\left(\frac{\pi}{2}\right)}}$

$$\left[2cis\left(\frac{\pi}{2}\right)\right]^{\frac{-1}{2}}$$ ✓Able to manipulate index

$$=\frac{\sqrt{2}}{2}cis\left(\frac{-\pi}{4}\right)$$ ✓Applys de Moivre's theorem

[2]

Question 2 [6 marks]

(a) (i) Find the quotient and the remainder for $\frac{z^3-2z^2+4z-1}{z^2-z+1}$, hence rewrite z^3-2z^2+4z-1 in the form $H(z)\times(z^2-z+1)+R(z)$ [3]

$$\frac{z^3-2z^2+4z-1}{z^2-z+1}$$

$$z^2-z+1\sqrt{z^3-2z^2+4z-1}$$ ✓ Divides to find H(z)

$$H(z)=z-1$$ ✓ Finds the remainder is 2z

$$(z-1)\times(z^2-z+1)+2z$$ ✓ rewrites the expression

(ii) Hence, solve $z^3-2z^2+4z-1=2z$ [3]

$$\frac{z^3-2z^2+4z-1}{z^2+z+1}=\frac{2z}{z^2+z+1}$$

$$(z-1)(z^2-z+1)=0$$

$$z=1$$ ✓ solves for z = 1

$$z^2-z+1=0$$

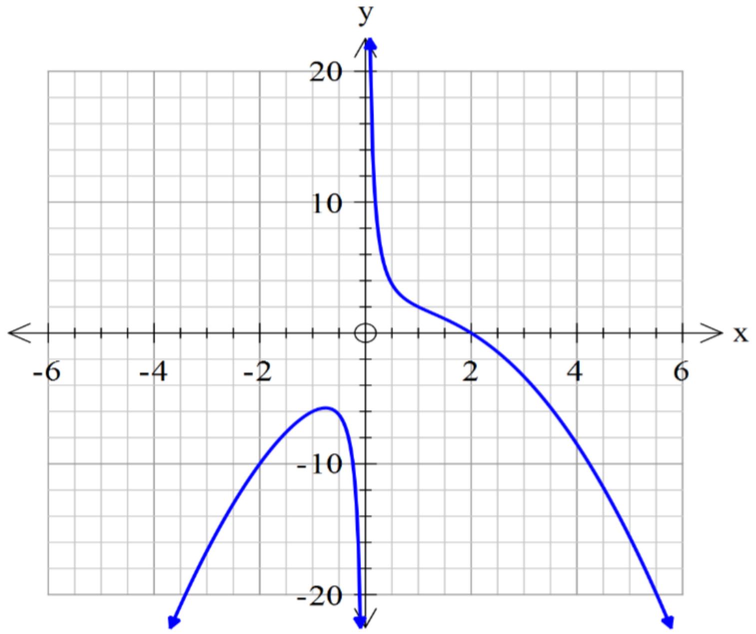
$$\left(z-\frac{1}{2}\right)^2-\frac{1}{4}+1=0$$ ✓✓ solves for the other two solutions

$$z=\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$$

$$z=1 \quad or \quad \frac{1}{2}\pm\frac{\sqrt{3}}{2}i$$

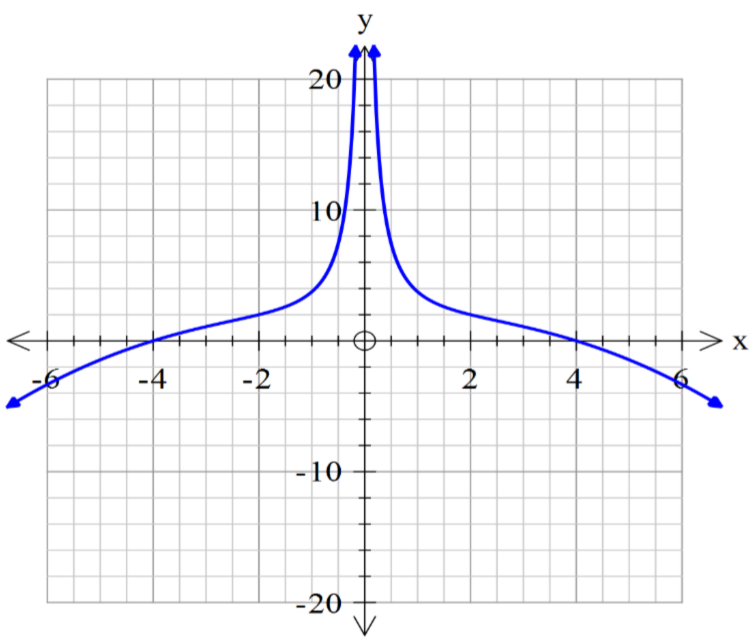
Question 3 [10 marks]

Given the graph of $y = f(x)$ is given as follows;



Sketch the graph of

(a) (i) $y = f\left(\left|\frac{x}{2}\right|\right)$



✓ removes the graph x<0

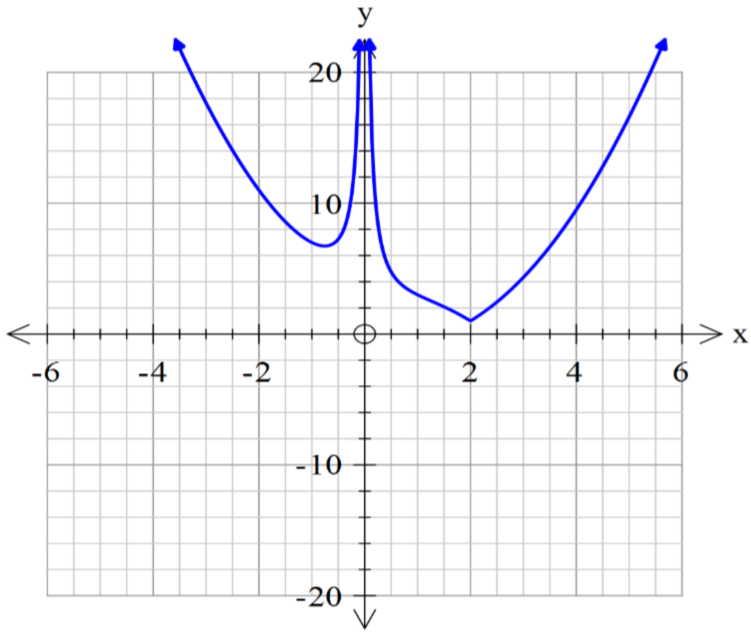
✓ mirrors the graph x>0 over the y-axis

✓ dilates the graph by a scale factor of 2 along the x-axis

[3]

(ii) Sketch the graph of $y = |f(x)| + 2$.

[3]



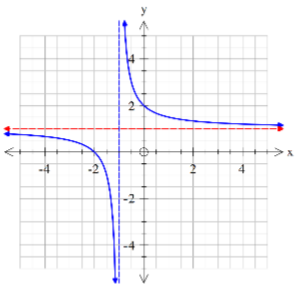
✓ reflects the part of the graph y<0 over the x-axis for x>0

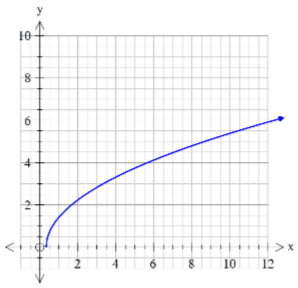
✓ reflects the part of the graph y<0 over the x-axis for x<0

✓ translates the graph 2 unit up

(b) Given that $g(x) = \sqrt{3x-1}$ and $h(x) = \frac{x+2}{x+1}$, find the domain and range of the composite function $goh(x)$

[4]





Domain		Range
$x > -1$ $x \leq \frac{-5}{2}$	$\geq \frac{1}{3}$ $\neq 1$	$y \geq 0$ $y \neq \sqrt{2}$

$\{x \in \Re; x \leq \frac{-5}{2}, \quad x > -1\}$

$\{y \in \Re; y \geq 0, \quad y \neq \sqrt{2}\}$

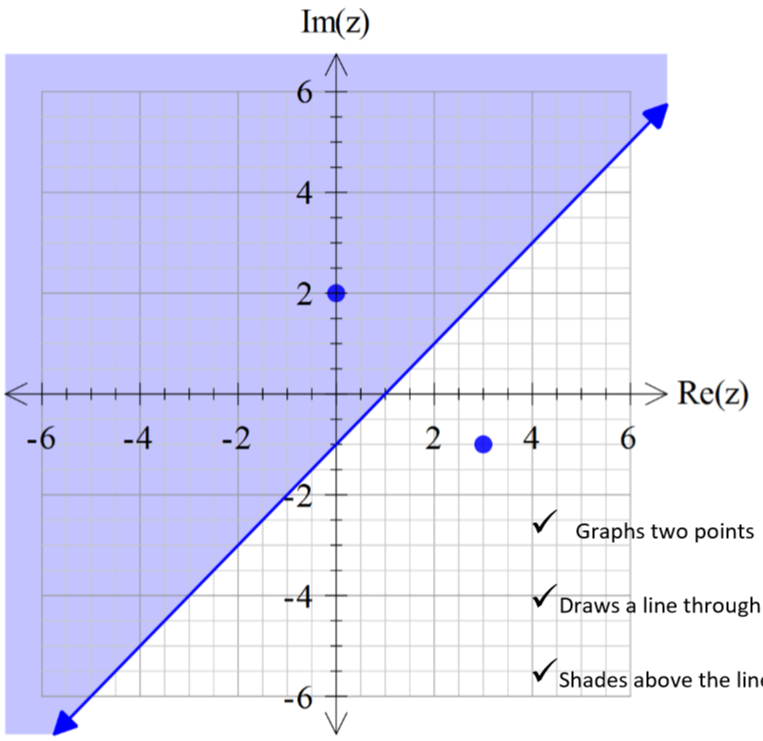
✓✓ Both domain restrictions (1 mark each)

✓✓ Both range restrictions (1 mark each)

Question 4 [5 marks]

(a) On an Argand diagram sketch the loci of points and that satisfy the following condition;

$$|z-2i| \le |z-3+i|$$



✓ Graphs two points

✓ Draws a line through the midpoint, perpendicular

✓ Shades above the line

[3]

(b) Give the equation of the locus in Cartesian form.

$$|z-2i| = |z-3+i|$$
$$x^2+(y-2)^2=(x-3)^2+(y+1)^2$$
$$-4y+4=-6x+9+2y+1$$
$$-6y=-6x+6$$
$$y \ge x-1$$

✓ Sets up Cartesian equation simplified

✓ Simplifies with correct inequality

[2]

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Calculator-assumed	2 minutes	25 minutes

Section Two (Calculator-assumed): 25 marks

Permissible items:

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators – CAS, graphic or scientific, satisfying the conditions set by the Curriculum Council for this course.

Write your answers in the spaces provided.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

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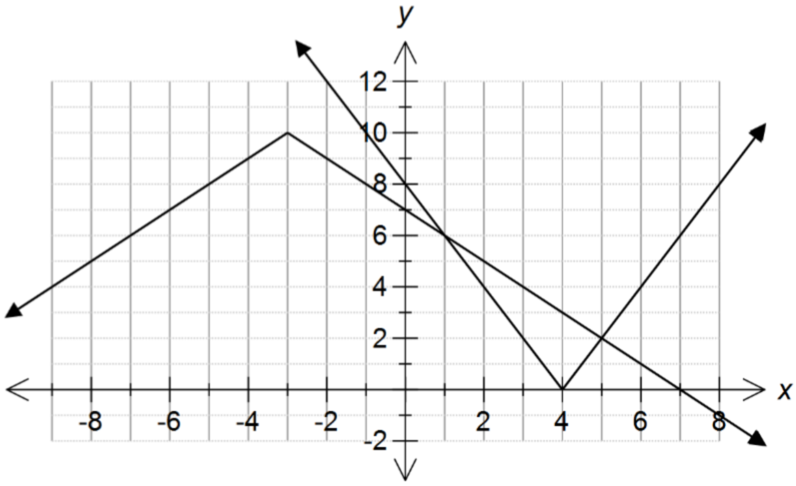
Section Two: Calculator-assumed **[25 marks]**

This section has **four (4)** questions. Answer **all** questions. Write your answers in the spaces provided

Question 5 [7 marks]



(a) Use the diagram above to solve for x in the following.

(i) $-|x+3|+10=7$ [1]

✓ Both x values given

$x=0 \;\; or \;\; -6$

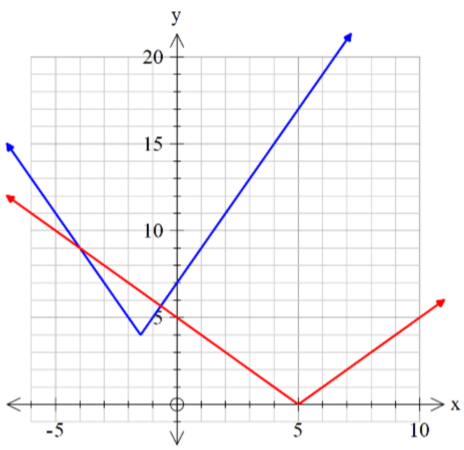
(ii) $-|x+3|+10 \geq |2x-8|$ [2]

✓ Correct x –values

✓ Correct domain

$1 \leq x \leq 5$

(b) Solve the following algebraically $4+|3+2x|>|x-5|$ [4]



$4+|3+2x|=|x-5|$

$4-3-2x=-x+5$

$1-x=5$

$x=-4$

$4+3+2x=-x+5$

$3x=-2$

$x=-\dfrac{2}{3}$

$-4>x>-\dfrac{2}{3}$

✓ Draws a graph and identifies the critical points of x=-1.5 and x=5

✓ Finds the correct linear equations of each relevant function

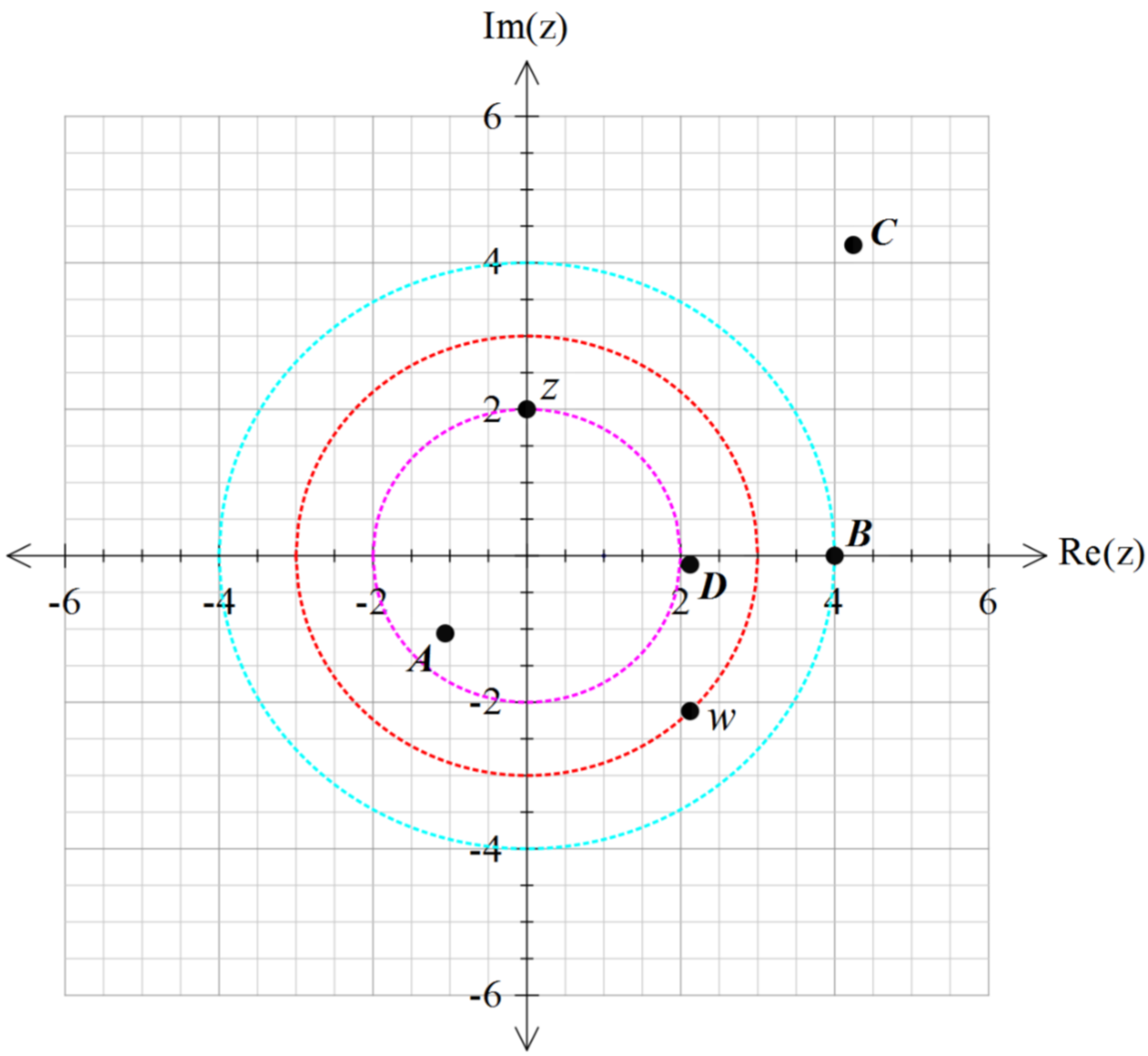
✓ Solves for the 2 intersections

✓ Writes the inequality correctly

Question 6 [4 marks]

Given the position of *z* and *w* on the Argand diagram below. Label the points A, B, C and D using the following options.

$w+z$ $\quad$ wz $\quad$ $\frac{-1}{2}w$ $\quad$ $z\bar{z}$ $\quad$ $\frac{w}{z}$ $\quad$ w^{-2} $\quad$ z^2



A $\frac{w}{z}$ ____________________________ [1]

B $z\bar{z}$ ____________________________ [1]

✓✓ ✓ ✓ 1 mark each correct answer

C wz ____________________________ [1]

D $w+z$ ____________________________ [1]

Question 7 [7 marks]

(a) Represent on the Argand diagram provided below, the loci of points, that satisfy the following conditions;

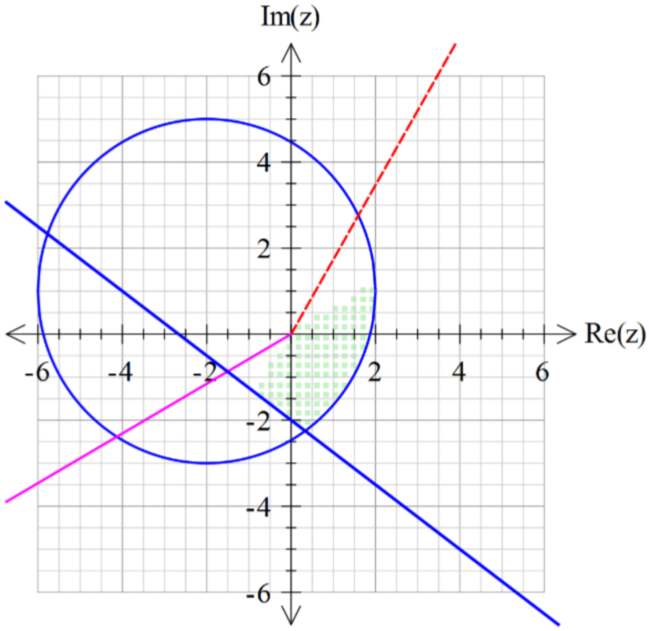
$$|z+2-i| \le 4 \quad , \qquad \frac{-5\pi}{6} \le \arg(z) < \frac{\pi}{3} \quad \text{and} \quad 4\,\text{Im}(z)+3\,\text{Re}(z)+8 \ge 0$$

✓ Circle drawn correctly

✓ Arg drawn correctly

✓ Line drawn correctly

✓ Shades correctly



[4]

(b) Given that $|z+2-i| \le 4$, state the minimum and maximum value of $|z|$.

$$|z+2-i| \le 4$$

✓ Finds radius

$$\sqrt{2^2+1^2}$$
$$=\sqrt{5}$$
$$\min|z| = 4-\sqrt{5}$$
$$\max|z| = 4+\sqrt{5}$$

✓ States min correctly

✓ States max correctly

[3]

Question 8 [7 marks]

(a) Using your CAS calculator (or otherwise) find all the solutions to $z^5 = 512\left(\sqrt{3}-i\right)$ in exact polar form, where $z = r(\cos\theta + i\sin\theta)$, $-\pi < \theta \leq \pi$ and $r \geq 0$.

[4]

$$z^5 = 512\left(\sqrt{3}-i\right)$$

$$z^5 = 1024cis\left(\frac{-\pi}{6}\right)$$

$$z_0 = 4cis\left(\frac{-\pi}{30}\right)$$

$$z_1 = 4cis\left(\frac{11\pi}{30}\right)$$

$$z_2 = 4cis\left(\frac{23\pi}{30}\right)$$

$$z_3 = 4cis\left(\frac{-25\pi}{30}\right)$$

$$z_4 = 4cis\left(\frac{-13\pi}{30}\right)$$

✓ Changes to polar form

✓ applies De Moivre's theorem and gives first solution

✓ Identifies they need to add 2PI/5

✓ Gives other 3 solutions

(b) Draw the solutions from (a) on the complex plane below. Show all major features.

[3]

✓ Correct placement and magnitude of first point z

✓ All other points magnitude of 4

✓ Other solutions are distributed evenly 2PI/5 angle apart